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by COLUMN RESEARCH COUNCIL

Memorandum on

"BASIC COLUMN STRENGTH"

(Proposed by Research Committee A)

3/30/55INTRODUCTION

The scope of the Council's assignment to Committee A is to determine the relation between material properties and the strength of columns. This is a proper preliminary to the consideration of other modifying factors that affect the strength of a real column in an engineering structure.

It is the purpose of this memorandum to make recommendations concerning the philosophy to be adopted for the design of axially loaded metal columns. The basic parameters are considered, including eccentricities, lateral loads, and end restraints. The memorandum is a technical contribution of Column Research Council's Committee A and the initial draft was prepared through the initiative and help of Dr. Lynn Beedle, who had made related studies as part of a Lehigh University Column Research Council Investigation. (1)

This memorandum relates to and builds upon two previous Column Research Council Memoranda: No. 1, on "The Basic Column Formula" (2) and No. 2 "Notes on Compression Testing". (3) It differs from these memoranda in that they were related to the evaluation of column strength, as determined by the compressive properties of the metal used, whereas this memorandum is related to the procedures to be used in the development of design formulas for any particular structural metal.

2. THE STRENGTH OF COLUMNS

The maximum strength of axially loaded columns is confirmed by test to be given approximately by an application in one form or other of the tangent-modulus concept. (2) Direct application is customary in the case of the aluminum alloys. (3) For structural steels containing residual stresses the strength of columns may be expressed in terms of the tangent modulus. As a result of its belief in the importance of residual stress as a factor in steel column design, Column Research Council has sponsored ^{research on this topic} ~~a pilot investigation~~ at Lehigh University.

The Pilot Investigation (1) demonstrated that residual stresses do affect column behavior resulting in a considerable lowering of column strength below values predicted on the basis of coupon tests, especially in the region of effective slenderness ratio of about 90. The Lehigh report also showed that due to residual stress, there is a pronounced difference in behavior dependent upon the axis about which the column bends. When a full cross-section is tested in the laboratory in compression the yield stress level may be markedly less than the tensile yield point determined in the routine ASTM acceptance test. This is due to variation in factors such as upper yield point, strain rate, and web strength vs flange strength. In generalized form the basic column strength formula is (2)

$$\frac{P}{A} = \frac{\pi^2 E \tau}{(KL/r)^2} \quad (1)$$

where τ is

$$\tau = \frac{E_t}{E}$$

τ represents the relative reduction in bending stiffness as gradual yielding takes place. Values of τ as a function of L/r (or of critical stress) and for flexure about the x- and y- axes could be developed easily. (1)

Eq. 2 was recommended by Bleich (4)

$$\sigma_c = \sigma_y - \frac{\sigma_p (\sigma_y - \sigma_p)}{\pi^2 E} \left(\frac{L}{r}\right)^2 \quad \dots (2)$$

in which

σ_c = average critical stress

σ_p = proportional limit

σ_y = yield stress

It is assumed that, because of residual stress, effectively, $\sigma_p = \sigma_y - \sigma_{rc}$, where σ_{rc} is the residual stress in compression at the flange corners. Very good correlation between theory and test is obtained by making indirect use of the tangent-modulus concept, as well as Bleich's approach. The following formulas were put forward (1) to express the basic strength of columns.

$$\sigma_{x-x} = \sigma_y - \frac{\sigma_{rc}}{\pi^2 E} (\sigma_y - \sigma_{rc}) \left(\frac{L}{r}\right)^2 \quad L/r < \pi \sqrt{\frac{E}{\sigma_y - \sigma_{rc}}} \quad (3a)$$

$$\sigma_{y-y} = \sigma_y - \frac{\sigma_{rc}}{\pi} \sqrt{\frac{(\sigma_y - \sigma_{rc})}{E}} \left(\frac{L}{r}\right) \quad L/r < \pi \sqrt{\frac{E}{\sigma_y - \sigma_{rc}}} \quad (3b)$$

$$\sigma = \frac{\pi^2 E}{(L/r)^2} \quad L/r > \pi \sqrt{\frac{E}{\sigma_y - \sigma_{rc}}} \quad (3c)$$

In the above equations "x-x" and "y-y" designate the flexure axes. σ_{rc} indicates residual stress at flange edges.

3. DEVELOPMENT OF WORKING FORMULAS

A considerable number of factors enter into the selection of working formulas for design use. Specific equations (or new "column formulas") can be put forward as recommended practice

when the parameters involved (σ_y , σ_{rc}) have been established for members in compression. It is considered, however, that the philosophy behind such equations may be advanced and some tentative values explored at this time.

(1) Yield Stress Level

The yield stress level will almost without exception exceed the "specification" yield point of 33,000 psi. In the current tests of full cross-sections an average basic compressive strength of material rolled to this specification (ASTM -A7) was found to be 37,000 psi. A statistical mean should be established for basic compressive strength and design formulas should then be based upon it.*

The factor of safety is intended to cover, among other things, under-run in dimensions and physical properties and on this basis, would cover those rare cases where the material just met the specifications. The alternate is to use the "minimum" value, and reduce the required factor of safety accordingly. It is emphasized, however, that this minimum value presently is no better defined than the average value. Use of the average value for design with the factor of safety to cover "underrun in physical properties" is considered the more rational approach. This

* For illustration, with the average material properties measured in the pilot program tests, Equations (3) become

$$\sigma_{x-x} = 37,000 - 1.074 (L/r)^2 \quad \dots (4a)$$

$$\sigma_{y-y} = 37,000 - 118.2 \frac{L}{r} \quad L/r < 110 \quad \dots (4b)$$

$$\sigma = \frac{290,000,000}{(L/r)^2} \quad L/r > 110 \quad \dots (4c)$$

These equations are plotted in Figure 1.

average value, of course, is that actually obtained from material rolled to a particular acceptance specification.

In Figure 4 is shown an average statistical curve representing over 3000 mill tension tests, supplied through the courtesy of Jackson and Moreland, Consulting Engineers, which indicates an average value of somewhere around 39,600 psi for the yield stress level. As has been noted (1) work is needed to establish this statistical average for compression specimens.

(2) Factor of Safety

Bleich in Reference 4 suggests on page 56 that a constant factor of safety be employed for all slenderness ratios.

This factor of safety should be of somewhat greater magnitude than that used for tension members.

"The considerations which determine the factor of safety fall into two groups: Unintentional variation of the loading condition, inefficiency of design methods, deviation of the cross-sectional areas of the members from the assumed values, etc., apply to all parts of a structure and therefore affect short and slender columns equally, while accidental imperfections, deviation of actual properties of material from the assumed standard, inaccurate estimate of the degree of fixity at the ends, effect of secondary stresses, etc., are factors which are closely related to the column problem and may have different weights depending on whether a short or slender column is concerned.

"The effect of unintentional eccentricity of axial load and also the effect of deviation of the column axis from the straight line are considerable for short and medium-length columns. Variation in characteristics of the material, especially of the yield point, influences column strength in the inelastic range appreciably, whereas slender columns remain unaffected by these variations, as their carrying capacity depends upon the modulus of elasticity, which varies but slightly from its standard value. On the other hand, an error in estimating the free length of a column is of effect on the calculated strength within the elastic range and of relatively slight influence in the inelastic range.

"In view of these facts no good reason seems to exist for designing short columns with a factor of safety lower than that applied in the cases of slender columns. Considering further all the uncertainties in the entire reasoning connected with the determination of the factor of safety, it appears advisable to rely upon an invariable value of this factor applying over the entire range of practical slenderness ratios."

By comparing Equations 4 with AISC and AREA - AASHO formulas for centrally loaded columns (1), it was determined that the minimum factor of safety was 1.90 for the present AISC formula. See Figure 5.*

The value of 1.90 should, of course, be examined and it may be desirable to raise or to lower this value. It is agreed that the factor of safety should be higher for compression members than for tension or flexural members. This is because the consequence of exceeding the working load ^{X F} may be much more disastrous in the case of columns than for tension or flexural members.

The value 1.90 is based on maximum strength. By comparison, the present AREA specification uses a factor of safety of

* For illustration, only, a hypothetical working formula, using $F = 1.90$, would take the form.

$$\sigma_{x-x} = 19,500 - 0.565 (L/r)^2 \quad (L/r < 110) \quad \dots (5a)$$

$$\sigma_{y-y} = 19,500 - 62.2 (L/r) \quad (L/r < 110) \quad \dots (5b)$$

$$\sigma_{ave} = \frac{153,000,000}{(L/r)^2} \quad (L/r > 110) \quad \dots (5c)$$

A comparison of Eqs. 5 with current formulas is given in Figure 2. Eqs. 5a and 5b offer no startling departure from the present practice insofar as the AISC Formula is concerned. The figure does indicate that some savings may be realized in column design if Eq. (5a) is used when columns are sufficiently braced so that flexure is about the strong axis.

It is significant that, even though as-delivered columns have an average stress-strain diagram as shown in Figure 3(a), the resulting equation does not require a reduction in allowable stress in comparison with present formulas which are based on the stress-strain behavior shown in Figure 3(b).

1.76 based on initial yield of an eccentrically loaded column in single curvature.

(3) Complexity

For the most efficient utilization of material, two curves are suggested by Eqs. (5a) and (5b) Figure 2): One is for the strong axis and one for the weak axis. If columns are sufficiently braced so that the maximum slenderness ratio is for the strong axis, then the upper curve could be used and an additional savings would be possible.

The following comment was made by Jonathan Jones, Chairman of the Practical Applications Committee of the Council:

"Past conditions have permitted us to be somewhat prodigal of materials, but that picture is changing. Materials obviously are going to grow more and more expensive, and their employment without waste is becoming of greater importance. In the design of compression members we have to a large extent coasted along, using rules and formulas derived from simple experiments made, and deductions drawn, long ago. We have established the rules either for average cases, meaning that some structural elements are not as well protected as they should be, or we have established them for the "worst case", thus wasting material in cases less severe."

In general it is suggested that a single solution be presented (weak axis - similar to Eq. (5b)), at the same time allowing the designer who wishes to do so to use an alternate (similar to Eq. 5a) when justified. This could be presented in an appendix to a specification. By this means a design which can afford the material need not be delayed by the necessity to consider a complicating factor.

(4) Eccentricity and Crookedness (Accidental)

It is suggested that the factor of safety be such that it include the effect of accidental eccentricity and crookedness.

A factor of safety of 1.90 is possibly sufficient to do this. It was noted above that Bleich suggests that the factor of safety include "unintentional variation of loading condition".

In the past it was common to explain the reduction in column strength in a region up to $L/r = 100$ as due to accidental eccentricities and initial curvature. Such accidental eccentricities, in fact, were estimated ^{in the past} and appear in the ^{certain design} ~~AREA~~ formulas, for example, in the term $ec/r^2 = 0.25$. Although the secant-type formula is derived on the basis of an idealized stress-strain curve with yield point at the elastic limit (33,000 psi), it assumes a certain value for accidental eccentricity. This latter value was arrived at analytically by correlation with a study of column tests. Since any correlating column tests must have included as-delivered specimens that contained residual stresses, the magnitude of the accidental eccentricity or initial curvature must necessarily have been arbitrary since a considerable portion of the reduction in column strength is now known to be due to the presence of residual stresses rather than eccentricities.

Reduction of column strength due to the combined influence of eccentricities and residual stresses has been studied in a current Lehigh program. (5) The results applied to the 8WF31 shape show that only for a limited range of L/r and eccentricity is the secant-type formula a precise representation of column strength. It indicates that both eccentricities and residual stresses must be considered in arriving at a rational column formula intended to include known eccentricity as a factor.

If the effect of residual stress is to be included, the original basis to assume an accidental eccentricity of 0.25 ec/r^2 no longer exists. However, if it were to be included in

(A)
(for 2nd ed. 24)

the basic formula, (along with residual stress effects) the allowable working stress would need to be lowered below present allowable stresses. Figure 6 shows curves taken from Ref. 5 for the strong and weak axis for an assumed eccentricity of $0.25 \text{ ec}/r^2$, and for a residual stress rate $R_c = \frac{\sigma_{rc}}{\sigma_y} = 0.3$. It shows that for certain L/r-ratios the actual factor of safety would be less than assumed in present formulas. The difference is less significant for the strong axis than for the weak axis.

No reduction in working stress appears necessary. The assumed magnitude of "accidental eccentricity" may be reduced to a value much less than $\text{ec}/r^2 = 0.25$ (since it is now known that residual stresses are the primary cause of reductions formerly assigned to "accidental eccentricity"), and the factor of safety may be considered as covering any real accidental eccentricity and initial curvature.

A further matter to consider is that in actual structures a column loaded concentrically is almost non-existent. A small accidental eccentricity has a greater influence on the strength of a "centrally loaded" column than it has on a column when the known eccentricity (or bending moment) is already large and has been considered. This is shown in Figure 7 (5). (For example, at $L/r = 50$ the reductions in strength are about equal, but the increments of eccentricity are not equal, being in the ratios of 1 to 1.5, 2.5, and 5.0). Is it logical, then, to reduce basic column strength to take care of this small eccentricity when a larger eccentricity is to be added later which nearly wipes out the influence of what was formerly a large factor? There appears to be a double reduction in allowable stress by this process.

This is particularly true when one considers the interaction type of formula,

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1 \quad . . . (6)$$

Since the f_b term already includes a known eccentricity, then there seems little logic in this "double reduction" of allowable stress.

Another factor which enters is that columns are seldom encountered that are loaded in single curvature -- the type of loading upon which the secant formula is based, and as shown by the inset in Figure 6. More often the column is in double curvature - a condition in which the effect of "accidentals" is even less.

The work of Research Committee B, "Initial Eccentricities of Compression Elements", should be particularly helpful when consideration is later given to members with bolted or riveted splices.

(5) Known Eccentricity (or Moments)

Committee A's assignment is limited to determining basic strength in compression; consequently the problem of known eccentricities and known moments is beyond its scope. The recommendation of Committee A is concerned with the limiting value when known eccentricities and moments are equal to zero.

It is hoped that the work of Research Committee D ~~at~~ ^{Penn State} Cornell, New York University, and elsewhere will lead to positive design recommendations with regard to columns with known eccentricities and lateral loads. The problem is not completely solved.

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(6) Restraints

This is another "Problem" beyond the scope of Committee A (it belongs with Committee D) but it does enter into a consideration of the factor of safety.

The curves of Figure 1 have been re-plotted in Figure 9 and labelled "Pin ends". If the ends were completely fixed, columns of the same slenderness ratio would be stronger, as indicated by the curves marked, "Full Fixity". The expression for critical stress for restrained columns is

$$\frac{P}{A} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2} \quad . . . (7)$$

in which K varies from 0.5 for fully fixed to 1.0 for pin ends, and to greater values in certain cases. This figure illustrates the reserve of strength due to end restraints for axially loaded columns. The reserve increases with increasing slenderness ratio to a maximum of 4 at $L/r = 220$ ^{and beyond} (L = full length of column).

Since most practical columns contain end restraints, this reserve acts to offset the influence of accidental eccentricities.

For restrained columns bent initially in single curvature the reserve of strength beyond the elastic limit is even greater than that indicated in Figure 8 (7). It therefore appears quite justified to let the factor of safety cover the effect of accidental eccentricity and crookedness in view of a larger reserve of strength due to "accidental" end restraint that is presently neglected.

For illustration, Figure 10 shows the column curve of Eq. 5b (a hypothetical working formula) and the maximum strength

column curve (Eq. 4b) upon which it is based.

With regard to eccentricities and end restraints, what is the worst situation that may arise? This is shown by the lower limit and represents single curvature with no restraint, an accidental eccentricity and crookedness, of $ec/r^2 = 0.1^*$ and a residual stress ratio ($R_c = \frac{\sigma_{rc}}{\sigma_y}$) of 0.3 (6). The minimum factor of safety is $\frac{20.7}{12.0} = 1.59$.

How frequently would this situation arise? Most infrequently. Columns in single curvature are extremely rare (the stronger "double curvature" is the usual case) and, secondly there are usually larger eccentricities (or lateral loads) which almost completely mask the influence of small eccentricities when considered separately.

The upper curve is for a centrally loaded column in which the end restraint is assumed sufficient to make the effective length factor, K, equal to .875 and it would be most unlikely to experience a combination of zero restraint and maximum accidental eccentricity.

Since the basic column formula will in most cases be used in conjunction either with an interaction formula of a type similar to Eq. (6) (or by some alternate method of handling large eccentricities), then the suggested procedure seems reasonable.

(7) Stress Condition

Current specifications, although derived, in large part, from a consideration of elastic behavior, actually depend in many

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*The influence of residual stress is taken here as equivalent to an $ec/r^2 = 0.15$ used in the secant formula.

cases on plastic action of the material in order to realize loads assumed in design. Common examples are (1) the assumption that the point of contra-flexure in tier building columns is at midheight in a wind analysis, and, (2) neglecting overstress in rivets and designing on the average shear of the group.

The tangent-modulus formula

$$\sigma_{cr} = \frac{\pi^2 E_t}{(L/r)^2} \quad . . . (8)$$

is based on a concept that allows for some inelastic deformation, and the design is based on the maximum load-carrying capacity.

Indeed, if the design philosophy were to be based on the premise that the yield point never be reached at maximum load, then the present factor of safety for as-delivered columns would only be of the order of 1.20. This is because residual stress would bring about yielding at the flange tips when the load was but 2/3 of the value at "nominal" initial yield.

Obviously, there is no need to require a regression from, perhaps, an "unintentional" consideration of ultimate strength as the design criterion.

4. FURTHER WORK

As mentioned, the basic parameters must be substantiated. This involves, for A7 steel, an evaluation of the yield stress of full cross sections in compression (σ_y taken as 37,000 in this memorandum) and the magnitude of compressive residual stresses at flange tips (σ_{rc} taken as 13,000 in this report).

For the other (high strength) steels additional work is required along the same line, but it is anticipated that for

rolled WF shapes the relationships will be similar.

Although it is expected that "cold-bending" of columns will be of less pronounced influence than cooling residual stresses, the influence of fabrication residual stresses must be followed through.

These and other phases of this general problem are being tackled by Research Committee A in their project on "Residual Stress and the Compressive Properties of Steel" at Lehigh University.

It is considered appropriate at this time, however, for Committee A to ask the Recommended Practices Committee for an opinion with regard to design recommendations. The final section of this memorandum puts forth some specific ^{suggestions} ~~recommendations~~.

5. RECOMMENDATIONS

The Column Research Council has assigned to Committee A the task of determining the relation between material properties and the strength of columns. The committee's work has now reached the stage at which recommendations may be made as to a "philosophy of design" wherein are concerned axially loaded columns of WF shape rolled to ASTM-A7 specification. The following is therefore recommended:

1. The basic strength of columns (without intentional eccentricities or lateral loads) is given by the tangent-modulus concept and forms the basis for design. The tangent-modulus formula is given by:

$$\frac{P}{A} = \frac{\pi^2 E_t}{\frac{(KL)^2}{r^2}}$$

An approximation to maximum strength for A7 columns of WF shape

based on the tangent-modulus concept would take the form*

$$\sigma_{cr}(x) = \sigma_y \frac{\sigma_{rc}}{\pi^2 E} (\sigma_y - \sigma_{rc}) \left(\frac{L}{r}\right)^2 \quad L/r < \pi \sqrt{\frac{E}{(\sigma_y - \sigma_{rc})}} \quad \dots (3a)$$

$$\sigma_{cr}(y) = \sigma_y - \frac{\sigma_{rc}}{\pi} \sqrt{\frac{(\sigma_y - \sigma_{rc})}{E}} \left(\frac{L}{r}\right) \quad L/r < \pi \sqrt{\frac{E}{(\sigma_y - \sigma_{rc})}} \quad \dots (3b)$$

$$\sigma_{cr} = \frac{\pi^2 E}{(L/r)^2} \quad L/r > \frac{E}{(\sigma_y - \sigma_{rc})} \quad \dots (3c)$$

2. Where design time will allow and where appropriate conditions of restraint exist, an appendix to the specification should allow for the economy resulting from a consideration of flexure about the strong axis.

3. The recommendations by Bleich that a constant factor of safety be employed for all slenderness ratios is endorsed. It is recommended that a study of an appropriate value for the factor of safety be undertaken.

4. The magnitude of the yield stress level σ_y (Eq. 3a, 3b) should be determined from the statistical mean (average) of compressive yield stress (instead of the specification "minimum" of 33,000 psi). (An average value of the residual stress magnitude (σ_{rc} is similarly required, and studies are continuing to determine a dependable value.)

* Based on work completed to date, these equations take the following form for A7 steel rolled to WF shapes:

$$\sigma_{cr}(x) = 37,000 - 1.074 \left(\frac{L}{r}\right)^2 \quad L/r < 110 \quad \dots (4a)$$

$$\sigma_{cr}(y) = 37,000 - 118.2 \left(\frac{L}{r}\right)^2 \quad L/r < 110 \quad \dots (4b)$$

$$\sigma_{cr} = \frac{290,000,000}{(L/r)^2} \quad L/r > 110 \quad \dots (4c)$$

5. Accidental eccentricity and crookedness are intended to be covered by the factor of safety in 3, above.

6. Allowable column load tables would be based on formulas of which Eqs. 5b and 5c are typical. (See Figure 2). Such equations would also be used as " F_A " in the interaction formula, Eq. 6. Accidental eccentricities and initial curvatures are covered in ^{the} formulas. not clear

7. Similar expressions would obtain for bridge specifications with reduction ~~as allowable~~ as agreed upon.

Factor of safety

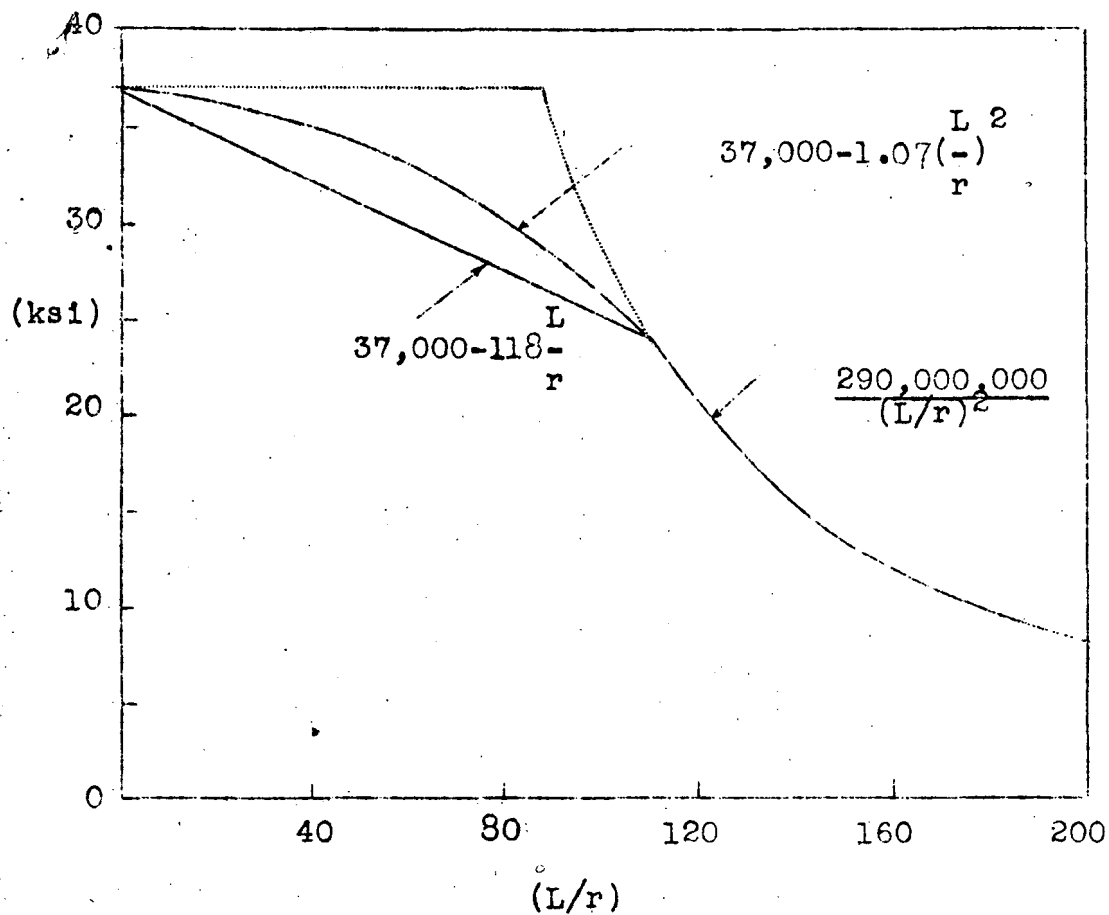


FIG.1
Maximum
Strength of
as-delivered
WF columns
(ASTM-A7)

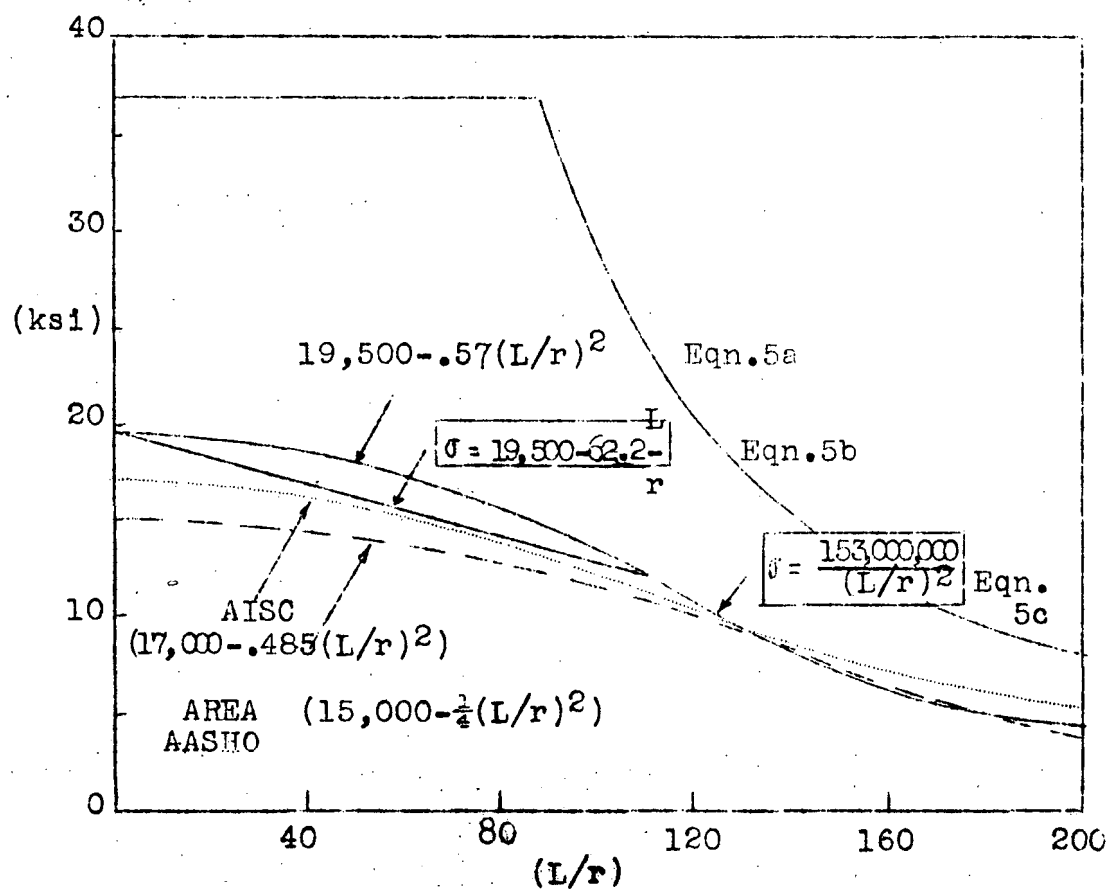
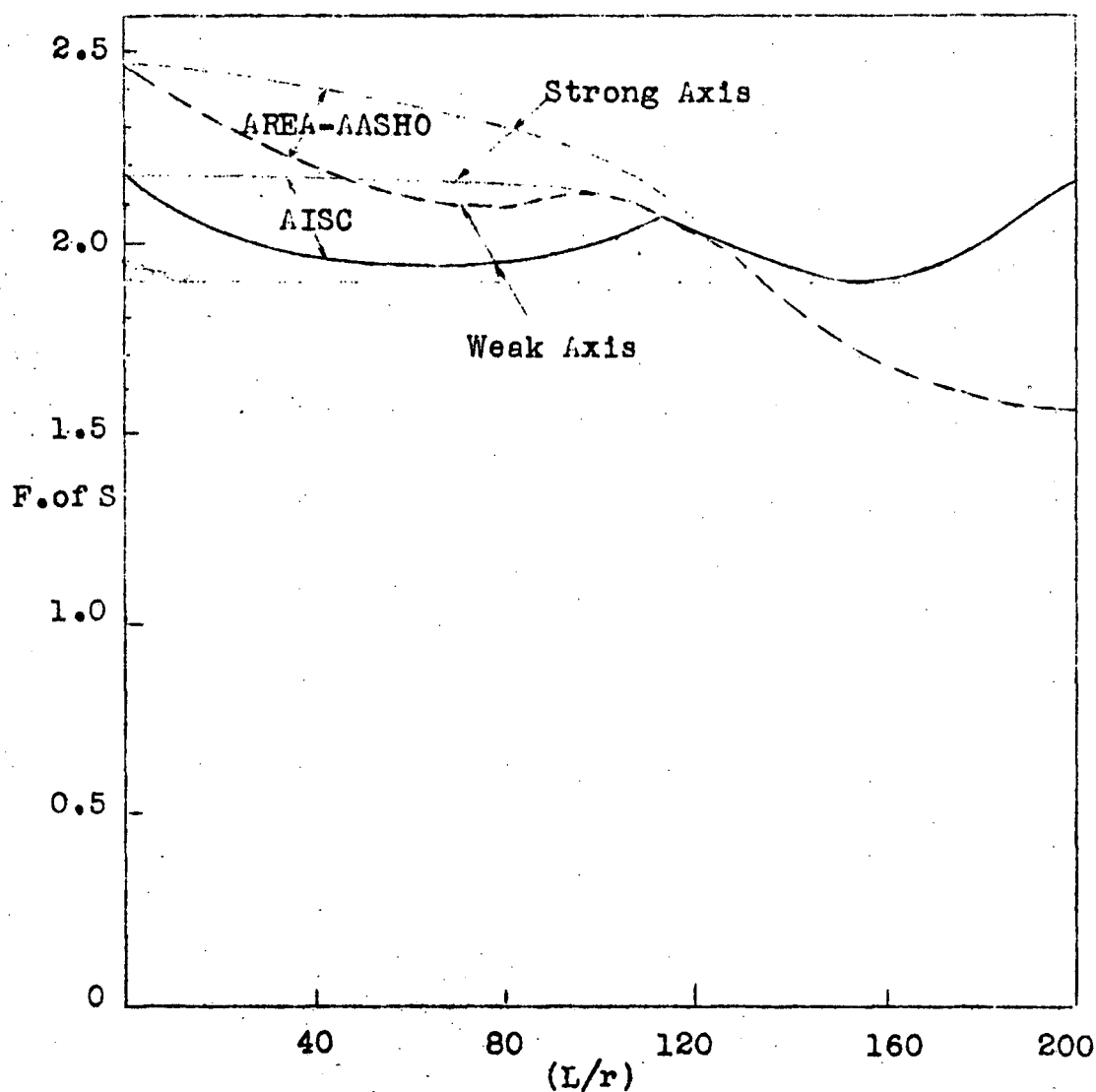
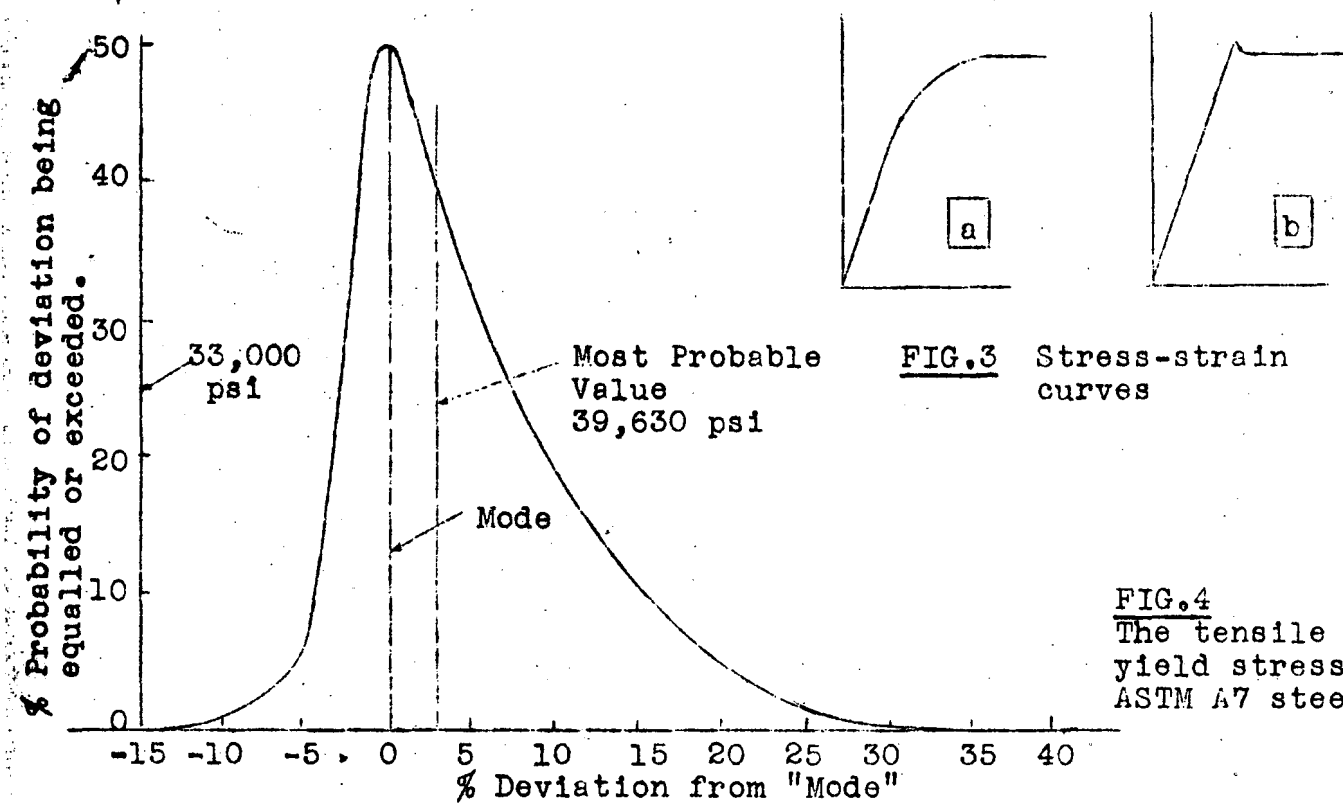


FIG.2
Comparison
with
Specifica-
tions.



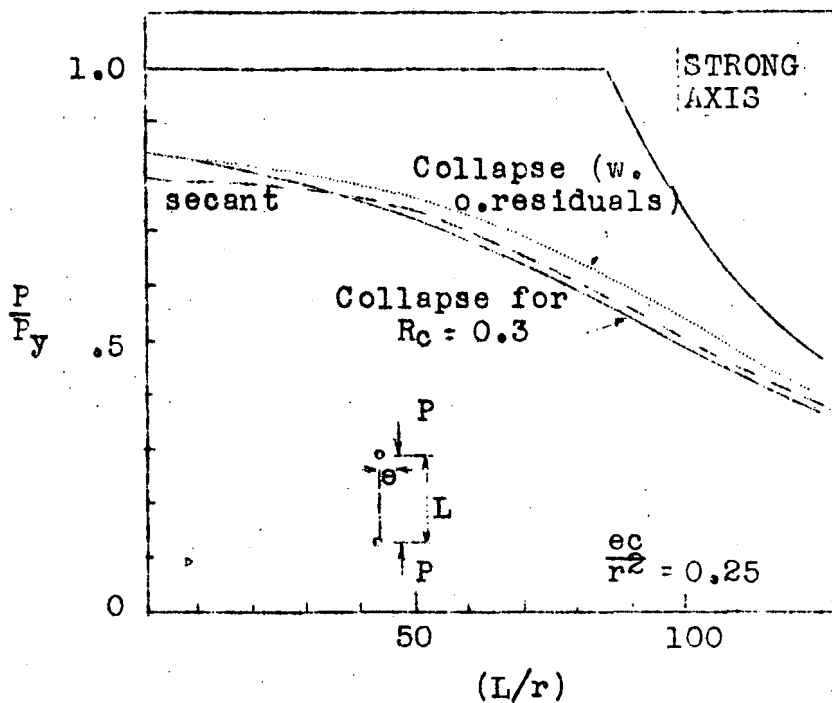


FIG. 6a
The strength of
eccentrically
loaded WF columns
at $\frac{ec}{r^2} = 0.25$

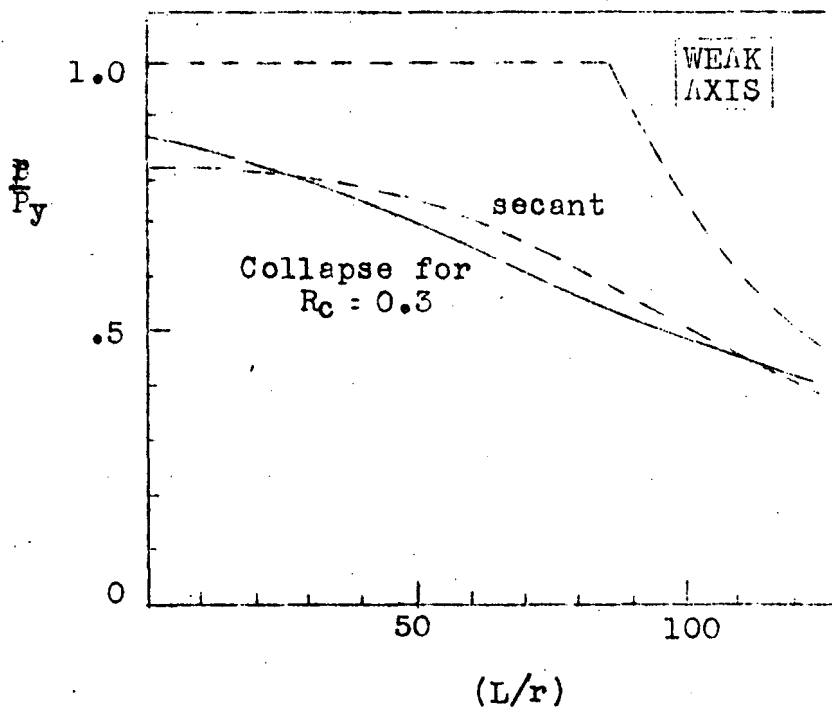


FIG. 6b
The strength of
eccentrically
loaded WF columns
at $\frac{ec}{r^2} = 0.25, R_c = \frac{C_{rc}}{\phi}$

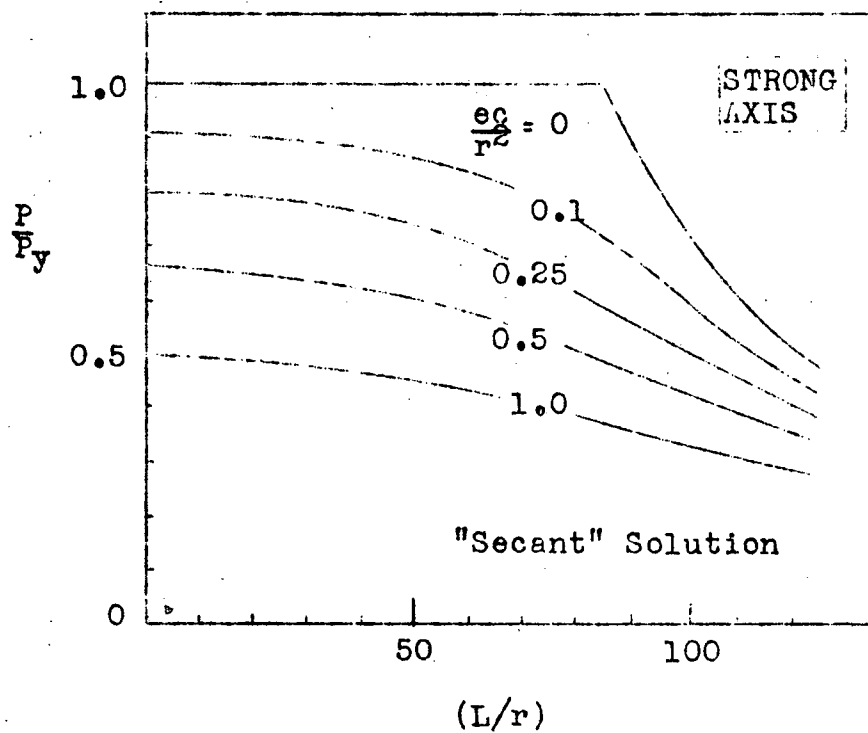


FIG.7⁽⁶⁾

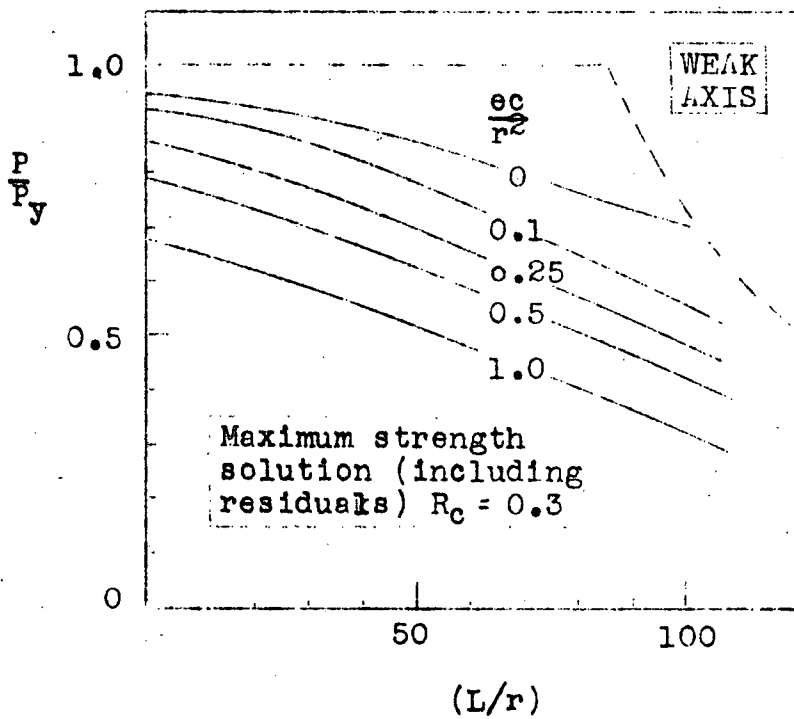


FIG.8⁽⁶⁾
Maximum strength
of eccentrically
loaded columns
containing
residual stresses.

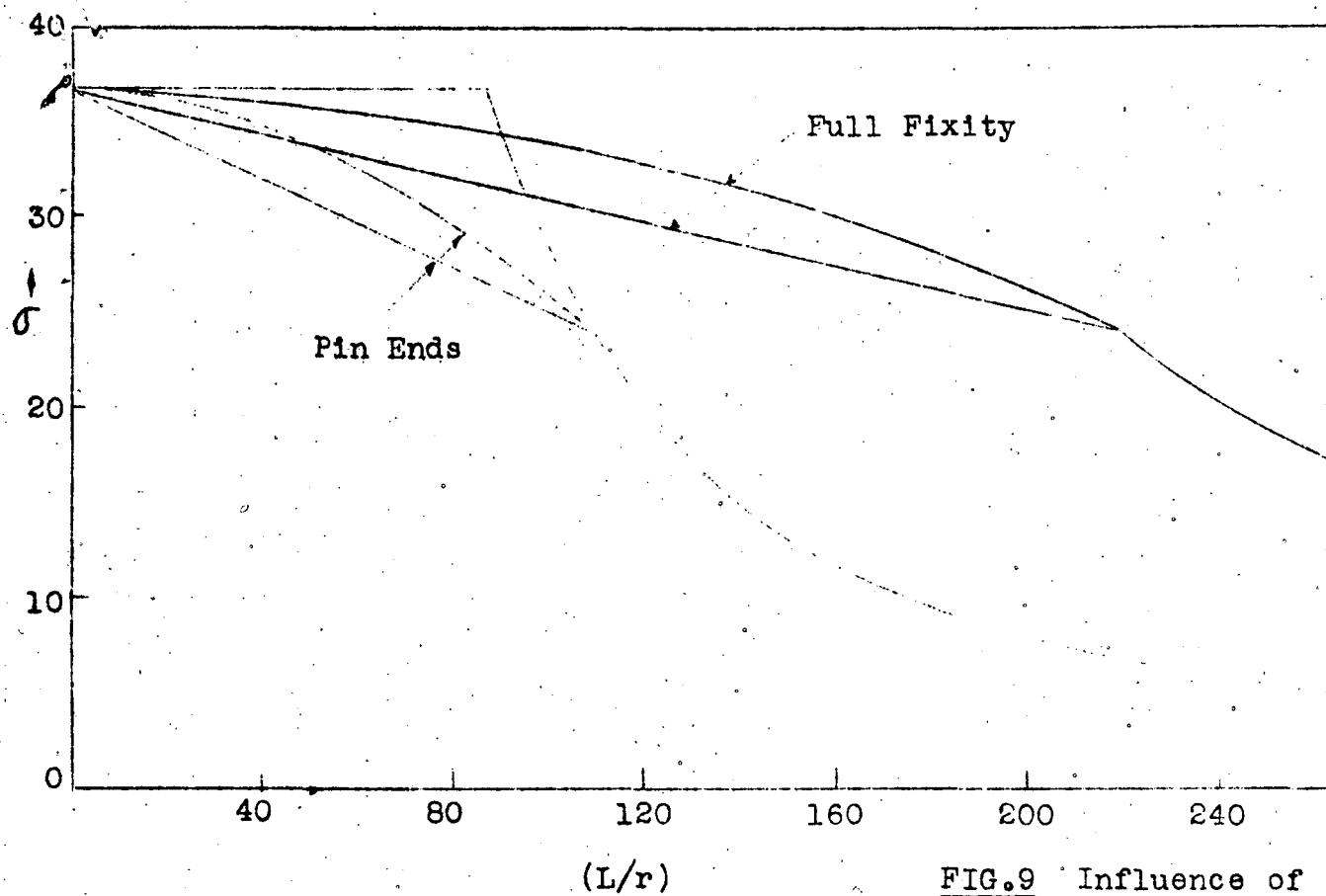


FIG.9 Influence of end restraints

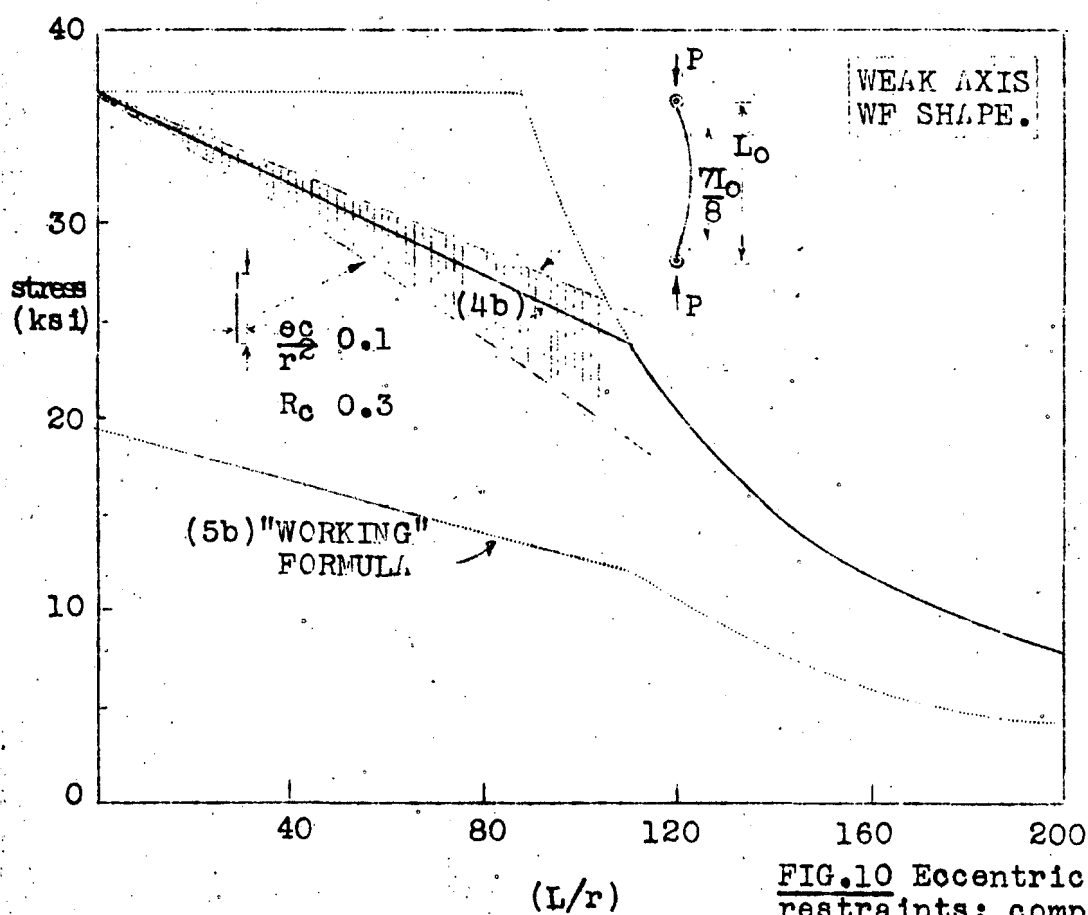


FIG.10 Eccentricities and restraints: comparison with Eq.4b and 5b.

now published

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